Practicum Three

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2 Replicating Nerlove's Classic Results on Scale Economies

The purpose of this exercise is replicate some of the principal returns to scale results reported by Nerlove in his classic 1955 article. The equation estimated by Nerlove is as follows:

$$lnC* = \beta_0 + \beta_y ln(y) + \beta_1 ln(p_1^*) + \beta_2 ln(p_2^*)$$

The data file NERLOV contains information on total costs (COSTS) in millions of dollars, output(KWH) in billions of kilowatt hours, and prices of labor (PL), fuels (PF), and capital (PK) for 145 electric utility companies in 1955. There are 145 observations, and the observations are ordered in size, observation 1 being the smallest company and observation 145 the largest.

a. Using the data transformation facilities of your computer software, generate the variables required to estimate parameters of Nerlove's equation. In particular, for each of the 145 companies, create the variables LNCP3 = ln(COSTS/PF), LNP13 = ln(PL/PF), LNP23 = ln(PK/PF), and LNKWH = ln(KWH). Print the entire data series for LNKWH, and verify that the observations are ordered by size of output, that is, that the first observation is the smallest output company and the last observations has the largest output.

```
## [1] "First five observations of the dataframe"

## # A tibble: 6 x 11
##     Obs ORDER  COSTS   KWH    PL    PF    PK LNCP3 LNP13 LNP23 LNKWH
##   <dbl> <dbl>  <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1     1   101 0.0820     2  2.09  17.9   183 -5.39 -2.15  2.32 0.693
## 2     2   102 0.661      3  2.05  35.1   174 -3.97 -2.84  1.60 1.10
## 3     3   103 0.990      4  2.05  35.1   171 -3.57 -2.84  1.58 1.39
## 4     4   104 0.315      4  1.83  32.2   166 -4.63 -2.87  1.64 1.39
## 5     5   105 0.197      5  2.12  28.6   233 -4.98 -2.60  2.10 1.61
## 6     6   106 0.0980     9  2.12  28.6   195 -5.68 -2.60  1.92 2.20

## [1] "Last five observations of the dataframe"

## # A tibble: 6 x 11
##     Obs ORDER COSTS   KWH    PL    PF    PK LNCP3 LNP13 LNP23 LNKWH
##   <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1   140   524  69.9  9484  2.11  24.4   165 1.05  -2.45  1.91  9.16
## 2   141   525  44.9  9956  1.68  28.8   203 0.444 -2.84  1.95  9.21
## 3   142   526  67.1 11477  2.24  26.5   151 0.929 -2.47  1.74  9.35
## 4   143   527  73.1 11796  2.12  28.6   148 0.938 -2.60  1.64  9.38
## 5   144   528 139.  14359  2.31  33.5   212 1.43  -2.67  1.85  9.57
## 6   145   529 120.  16719  2.30  23.6   162 1.63  -2.33  1.93  9.72
```

Note: Because the question requests printing the full dataset, I have appended a separate page of all observations to the end of this document. When compiling the markdown file, it will not print all observations of any dataset with this many records.

b. Given the data for all 145 firms from (a), estimate the following equation:

$$lnC* = \beta_0 + \beta_y ln(y) + \beta_1 ln(p_1^*) + \beta_2 ln(p_2^*)$$

where:

$$ln(CP3) = ln(C^*), ln(KWH) = ln(y), ln(P13) = ln(p_1^*), ln(P23) = ln(p_2^*)$$

Nerlove reported parameter estimates for By, B1 and B2 as 0.721,0.562 and -0.003, ,respectively, with standard errors of 0.175, 0.198, and 0.192, respectively, and an R2 of 0.931. Can you replicate Nerlove's results? (Note: You will not be able to replicate Nerlove's results precisely. One reason for this is that he used common rather than natural logarithms; however, this should affect only the estimated intercept term. According to Nerlove, the data set published with his article is apparently an earlier one that includes errors, while a revised data set was used in the estimation. This final data set has never been found.)

Table 1: Summary Statistics for Model One

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-4.6907892	0.8848713	-5.3010975	0.0000004
LNKWH	0.7206875	0.0174357	41.3339759	0.0000000
LNP13	0.5929096	0.2045722	2.8982907	0.0043523
LNP23	-0.0073810	0.1907356	-0.0386978	0.9691861

c. Using the estimates you obtained in part (b) and a reasonable level of significance, construct a confidence interval for By. Is the null hypothesis that By = 1 rejected? What does this imply concerning a test of the null hypothesis that returns to scale are constant? Using the relation between return to scale(r) and By (By = (1/r)) , compute the point estimate of returns to scale(r) based on your estimate of By. Are estimated returns to scale increasing, constant, or decreasing? Are economies of scale positive, zero, or negative?

Formula for a confidence interval (using a significance level of 95%)

$$B_y \pm 1.96(SE(B_y))$$

$$0.720688 \pm (1.96)(0.017436)$$

Confidence interval: (0.6865134, 0.7548626)

Explanation: Based on the confidence interval that is given above, the statistical decision is to reject the null hypothesis that B_y = 1 at the 95% threshold because the value of 1 does not lie within the confidence interval.

Returns to scale are calculated as follow:

r = 1/B_y

r = (1/0.720688)

r = 1.387563

Based on the point estimate **returns to scale = 1.387563**, the returns to scale are increasing because r > 1. This means that the firms in the dataset have increasing returns to scale in their energy production. There are **positive economies of scale** for energy production.

d. Demands for each factor of production will be positive only if ai, the coefficient in the Cobb-Douglas production function, is positive, i= 1, 2, 3. Note that we can link the estimated parameters from the regressions above to the parameters of the Cobb-Douglas as follows:

$$\alpha_1 = \beta_1(r)$$

$$\alpha_2 = \beta_2(r)$$

What is the implied estimate of alpha 2 from part (c)? Is it significantly different from zero? Why do you think Nerlove was unsatisfied with this estimate of alpha 2?

Calculating the alphas:

$$\alpha_1 = \beta_1(r)$$

$$\alpha_1 = (0.592910)(1.387563)$$

$$\alpha_1 = 0.8227$$

$$\alpha_2 = \beta_2(r)$$

$$\alpha_2 = (-0.007381)(1.387563)$$

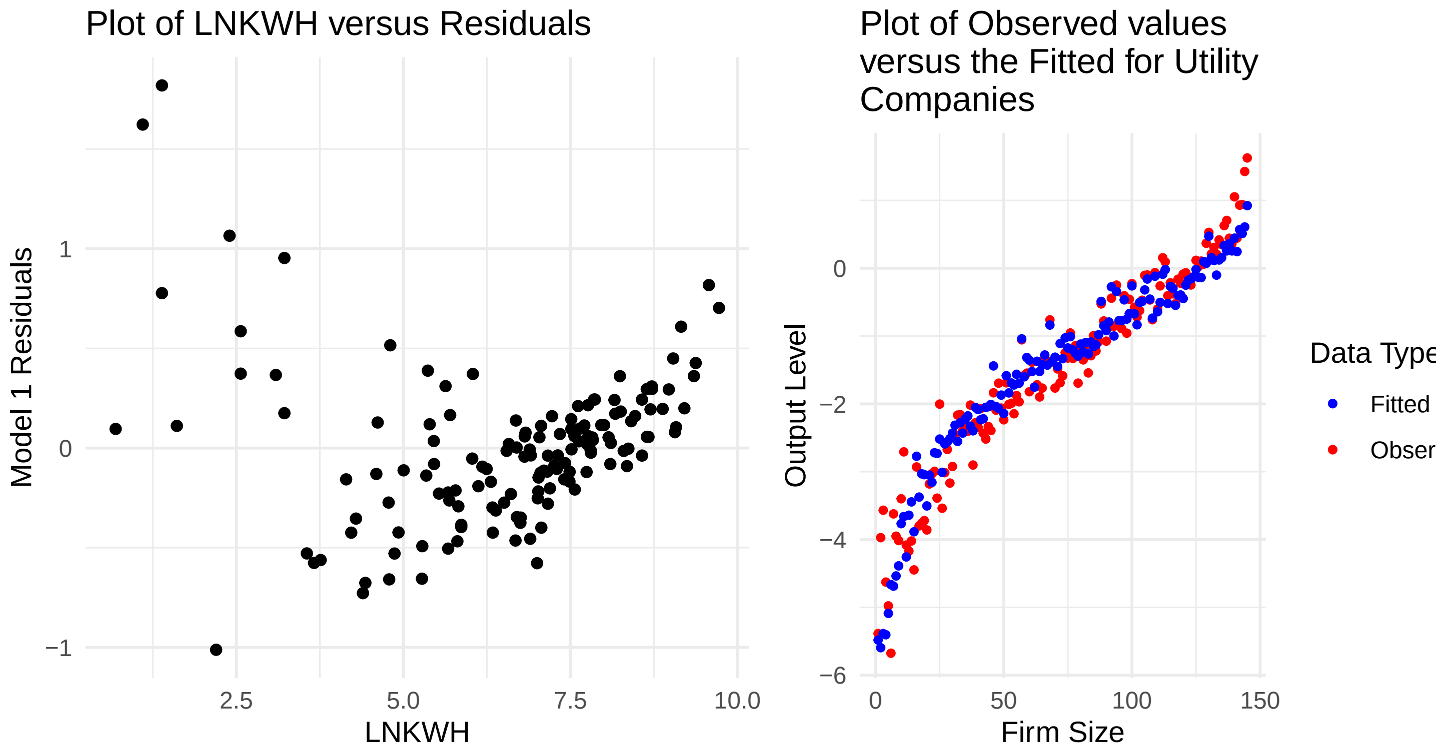
$$\alpha_2 = -0.0102416$$

Interpretation:

The implied estimate is alpha 2 = -0.010. From the regression output, we can assess whether or not it is significantly different from zero using the t-test. The summary statistic table given in part b states for the beta 2 coefficient used to calculate alpha 2 that t = -0.039, because |t| < 1.96, we reject the null hypothesis that beta 2 is significantly different from zero with 95% confidence. The beta 2 coefficient **is not significantly different from zero**, and the same can then be said for the alpha 2 coefficient.

Nerlove was unsatisfied with this estimate of alpha 2 based on the following line of reasoning. The beta 2 coefficient measures the percent change in cost associated with the percent change in the ratio of the price of capital to fuel. Nerlove likely included this variable in the regression because he wanted to determine if the ratio of the price of capital and fuel inputs would have in input on cost. Given that fuel and capital (and their associated costs) are factors in energy production, it is reasonable to think that these factors would have a statistically significant impact on the total cost of the utility companies within this dataset. The implication of the regression output is that this is not the case, from the summary table and the analysis above, the Beta 2 coefficient (that measures the ratio of capital price to fuel price) does not have an impact on the cost for the utility company. This contradicts what we would believe intuitively.

e. Compute and plot the residuals from estimated Nerlove's regression equation. Nerlove noticed that if the residuals were plotted against the log of output, the pattern was U-shaped, residuals at small levels of output being positive, those at medium levels of output being negative, and those at larger levels of output again becoming positive. Do you find the same U-shaped pattern? How might this pattern of residuals be interpreted? Finally, what is the sample correlation of residuals with LNKWH across the entire sample? Why is this the case?

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## [1] -9.933073e-17
```

Questions related to the plot:

Examining this plot, my analysis also returns a U-shaped pattern for the log of output versus residuals. Looking at the pattern of residuals, the second graph helps to explain the pattern in which residuals are positive for the small and large utility companies and negative for the medium sized utility companies. The second graph compares the values fitted by the model (blue) versus the values observed in the dataset. We see that for the medium sized companies (whose output lies in the middle of the figure), the model is overestimating their output because fitted values > observed values. For the small and large companies (whose output lie on either extreme of the graph), the model is underestimating their output because fitted values < observed values. Although this trend is observed, it is likely not of serious consequence because the difference between fitted and observed is not very large.

Question regarding sample correlation)

The sample correlation of residuals with lNKWH across the entire sample is zero. This is likely the case because of what was described in the previous part. There are positive residuals for small and large utility companies; because these are at both extremes of the dataset, the correlation should be zero. Factoring in the negative residuals for medium sized utility companies (which would offset the positive correlation in the extremes), there is no apparent trend across utility company size that can be drawn between model residuals and the output across the utility companies (as given by the LNKWH variable).

3 Assessing Alternative Returns to Scale Specifications

Because of the pattern of residuals noted by Nerlove (see part (e) in the previous question), Nerlove hypothesized that estimated returns to scale varied with the level of output. In this exercise you evaluate Nerlove's conjecture and assess alternative specifications that relax the assumptions implicit in Nerlove's equation. To facilitate grouping the data in this exercise your data (NERLOV) contains a variable named ORDER; the first 29 values of this variable are numbered 101 to 129, the second set of 29 values are 201 to 229, and so forth, the final 29 values of the variable ORDER taking on the values 501 to 529.

Following Nerlove, divide the sample of 145 firms into five subsamples, each having 29 firms. Recall that since the data are ordered by level of output, the first 29 Observations will have the smallest output levels, whereas the last 29 Observations will have the largest output levels. Then using least squares regression techniques, estimate parameters of Nerlove's for each of these subsamples.Nerlove [1963. p. 176] reports the following results (estimated standard errors are in parentheses): How well can you replicate Nerlove's reported results? To what might you attribute any discrepancies? (Note: See the brief discussion at the end of part (b) of Exercise 2.)

Note: Beta coefficients and respective standard errors in this table are presented in the same order as in the model specification. B__0, B__y, B__1, B__2

Table 2: Summary Statistics for each Utility Company Group

Group	Coefficients	StandardError	Rsquared
1	-3.3433, 0.4003, 0.6152, -0.0814	(3.1457), (0.0845), (0.7293), (0.7064)	0.5134
2	-6.489, 0.6582, 0.0938, 0.3779	(1.4129), (0.1163), (0.2743), (0.2765)	0.6328
3	-7.3329, 0.9383, 0.4023, 0.25	(1.689), (0.198), (0.1994), (0.187)	0.5732
4	-6.546, 0.912, 0.507, 0.0934	(1.1648), (0.1075), (0.1875), (0.1641)	0.8726
5	-6.7143, 1.0444, 0.6026, -0.2894	(1.0463), (0.065), (0.1973), (0.1749)	0.9210

I will likely attribute the discrepancies to the fact that the regression models ran in this analysis use ln-natural log, whereas Nerlove's use a regular log.

b. On the basis of your parameter estimates of By in part (a), compute the point estimates of returns to scale (r) in each of the five subsamples. What is the general pattern of estimated scale economies as the level of output increases? How might this pattern be interpreted? Does this suggest an alternative specification?

Computing the returns to scale for each utility company size (denoted group 1-5 in the previous table summary statistics) Formula: Note: B^k, indicates the beta coefficient for each group, k = 1,. . . 5

$$r = 1/(\beta_y^k)$$

$$r_1 = 1/(0.4003) = \underline{2.498126}$$

$$r_2 = 1/(0.6582) = \underline{1.519295}$$

$$r_3 = 1/(0.9383) = \underline{1.065757}$$

$$r_4 = 1/(0.912) = \underline{1.096491}$$

$$r_5 = 1/(1.0444) = \underline{0.9574876}$$

Explanation: Examining the returns to scale across each group, we see that the lower group numbers (small utility companies) have higher returns to scale compared to the larger group numbers (large utility companies). *The pattern of returns of scale then, is that as the size of the utility company increases, the returns to scale diminish.* The interpretation of this is that smaller utility realize greater returns to scale compared to larger firms.

For economies of scale, the pattern shows that through the first four sub-groups of firms in the dataset, returns to scale are >1, and thus the companies enjoy economies of scale. However, the final group of utility companies have diseconomies of scale, the returns to scale are < 1 which we interpret as the size of the company is so large that cost per unit has increased. This may suggest an alternative specification if we believe that economies of scale should hold for this fifth group.

c. Now construct data variables such that Nerlove's equation will be estimated, except that while each of the five subsamples has common estimated "slope" coefficients for B1 and B2, each of the five subsamples has a different intercept term and a different estimate of By. Given the results in part (b), why might such a specification be plausible? Estimate this expanded model, and assess your success in replicating Nerlove, who reported the five subsample estimates ofBy, as being 0.394 (0.055), 0.651 (0.189), 0.877 (0.376), 0.908 (0.354), and 1.062 (0.169), respectively, where numbers in parentheses are standard errors. The common estimates of B1 and B2 reported by Nerlove are 0.435 (0.207) and 0.100 (0.196), respectively. Nerlove's reported R2 was 0.95.

Table 3: Summary Statistics for Model Three

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-4.1798237	0.7022278	-5.9522330	0.0000000
LNP13	0.4256102	0.1631745	2.6083135	0.0101395
LNP23	0.1037274	0.1522073	0.6814878	0.4967478
groupfac2	-0.8726116	0.8846828	-0.9863554	0.3257497
groupfac3	-2.4502971	2.0767251	-1.1798851	0.2401520
groupfac4	-2.5487879	2.1110670	-1.2073458	0.2294418
groupfac5	-3.9035342	1.1607069	-3.3630663	0.0010071
LNKWH	0.3968780	0.0430725	9.2141857	0.0000000
groupfac2:LNKWH	0.2512856	0.1534807	1.6372454	0.1039432
groupfac3:LNKWH	0.4878991	0.3007413	1.6223218	0.1071026
groupfac4:LNKWH	0.5118603	0.2771074	1.8471547	0.0669469
groupfac5:LNKWH	0.6658665	0.1384123	4.8107473	0.0000040

Why might this specification be plausible: This model specification treats the marginal effects (% change in variable associated with % change in cost) for the variables LNP13- the ratio of labor to fuel price and LNP23- the ratio of capital to fuel price as being the same across all different sizes of utility company. Then it treats the intercept term, as well as the slope term for LNKWH as being different for each company. The motivation behind doing so is a hypothesis that explaining that the marginal effect of a change in millions of kilowatt hours varies across the size of the company, though the effect of the LNP13 and LNP23 factors is constant across companies.

In the context of what was established in question two, that scale economies only exist up to a certain point, and then diseconomies of scale became apparent for the largest of utility companies within the dataset, this alternative specification gives more insight on why that was observed. Because the previous specifications in part b allowed all marginal effects to vary whereas the specification in part c only allows the marginal effect of LNKWH to vary, these two have different interpretations. It is possible that diseconomies of scale were apparent for the largest of firms in part b due to allowing the variables LNP13 and LNP23 to also vary across group size. *Explaining this in a more plain manner, it is not the ratio of labor to fuel price or ratio of capital to fuel price that are driving economies of scale, but perhaps the LNKWH. This is the motivation for running this model.* By holding these constant in the part C specification, these marginal effects are properly controlled for, and we can see by the increase in the B_y coefficient, that returns to scale are apparent, and actually increasing as the size of the utility company increases.

d. For each of the five subsample estimates of By in part (c), compute the implied estimate of returns to scale. What is the general pattern of estimated scale economies as the level of output increases?

Formula:

$$r = 1/B_y^k$$

$$r_1 = 1/0.39688 = \underline{2.51965}$$

$$r_2 = 1/0.64817 = \underline{1.54280}$$

$$r_3 = 1/0.88478 = \underline{1.130224}$$

$$r_4 = 1/0.90874 = \underline{1.10042}$$

$$r_5 = 1/1.06275 = \underline{0.94095}$$

These calculations contradict the theory established in the analysis from part c; LNKWH drives economies of scale in the same manner in the regression for part c (where LNP13 and LNP23 are constant across all utility company sizes) as that in regression for part b (where all variables are different across utility company sizes). **The interpretation of this result is that utility company size using output (millions of kilowatt hours) as a measure will yield increasing returns to scale, and economies of scale up to a threshold given by the output of the firms in group four, after which there are decreasing returns to scale, and diseconomies of scale.**

e. How would you compare estimates in part (c) versus those in part (a)? In particular, since part (a) estimates constitute a special case of part (c), using an F-test and a reasonable level of significance, formulate and test the restrictions implicit in part

(a) against the alternative hypothesis in part (c). Is the null hypothesis rejected or not rejected? Comment on your results.

e. In part (a), there are five regressions and thus five sets of summary statistics whereas model (c) only has one set of summary statistics. There are two potential approaches to measuring the improvement in model performance. First, we could compare each of the R^2 values in the unrestricted model (model a) to that of the restricted model (model c), the statistical significance of which would indicate the performance each unrestricted model (a) to that of the restricted model. Alternatively, and the approach I'll use is to take the average R^2 from the five models in model (a) and use that to compute the F test, comparing its model performance relative to that of the restricted model (c).

Average R^2 of model (a): = avg(0.5134,0.6328,0.5732,0.8726,0.9210) = 0.7026 R^2 of model (c): = 0.9602

F Test formula:

$$F = ((R_{ur}^2 - R_r^2)/(DF_r - DF_{ur}))/((1 - R_{ur}^2)/(n - k - 1))$$

$$F = ((0.7026 - 0.9602)/(-8))/((1 - 0.7026)/(132))$$

$$F = 14.2918$$

e. This F statistic shows the *on average* improvement of the model performance when all the variables (LNKWH, LNP13, and LNP23) are allowed to vary under the unrestricted model, compared to the performance of the restricted model that only allows for the intercept and the LNKWH to vary. A value of 14.2918 on 132 degrees of freedom will be statistically significant at all levels, but it is low enough that we cannot say the unrestricted models are a great improvement over the restricted model. Another thing to note is that this is the R^2 on average for the unrestricted models, this could potentially introduce upward bias for the models within the unrestricted set of models that have a lower R^2, and downward bias for those with a higher R^2.

This could be fixed by comparing the R^2 of each individual unrestricted model to the restricted (so, five calculations), though there is bias in this approach because the restricted model contains data from the entire group while the unrestricted is a subset with only 29 observations per group.

f. To exploit the fact that estimated returns to scale seemed to decline with the level of output in a nonlinear fashion, Nerlove formulated and estimated a slight generalization of the original equation in which the variable ln(y)^2 was added as a regressor; call the corresponding coefficient Byy. Using the full sample of 145 observations, estimate the equation:

$$ln(C^*) = \beta_0 + \beta_y ln(y) + \beta_{yy} * ln(y)^2 + \beta_1 ln(p_1^) + \beta_2 ln(p_2^*)$$

by least squares. How well can you replicate Nerlove's reported results, which he reported as 0.151 (0.062), 0.117 (0.012), 0.498 (0.161), and 0.062 (0.151) for By, Byy , B1, and B2, respectively, and an R2 of 0.952? Now, using a reasonable level of significance, test the joint null hypothesis that returns to scale are constant, that is, that By = 1, Byy = 0, against the null hypothesis that returns to scale are nonconstant, that is, thatBy != 1. Byy != 0. How does inference based on the joint F-test compare with that based on the individual t-tests? Finally, since returns to scale in the above expanded model vary with the level of output and can be shown to equal r = 1/(By + 2 × By × lny), compute the implied range of returns-to-scale estimates using the median value of LNY in each of the five subsamples.

Table 4: Summary Statistics for Model Four

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-3.7646483	0.7017266	-5.3648363	0.0000003
LNKWH	0.1525466	0.0618601	2.4659922	0.0148717
SQLNKWH	0.0505140	0.0053637	9.4178231	0.0000000
LNP13	0.4805858	0.1610724	2.9836629	0.0033615
LNP23	0.0741663	0.1500159	0.4943898	0.6218058

Table 5: Joint Null Hypothesis:

Res.Df	Df	F	Pr(>F)
142	NA	NA	NA
140	2	70.47526	0

f. The output from the linear hypothesis test provided above shows that we can reject the null hypothesis that B__y = 1 and B__yy = 0 at all reasonable levels of significance in favor of the alternative hypothesis that B__y != 1 and B__yy != 0. Justification: $P < 0.05$. Inference on the joint F-test is stronger compared to that of individual t-tests in that we are assessing the significance of multiple variables taken together in a regression rather than just individual variables. Assessing the individual variables would require assessing the statistical significance of each variable from the regression summary statistics.

Computing the returns to scale in the expanded model: Formula:

$$r = 1/(\beta_y + 2 * \beta_y * ln(y)$$

Group 1:

r = 1/(0.152547+2*(0.152547)+3.761200) = **0.2370319**

Group 2:

r = 1/(0.152547+2*(0.152547)+5.823046) = **0.1592183**

Group 3:

r = 1/(0.152547+2*(0.152547)+7.011214) = **0.1338893**

Group 4:

r = 1/(0.152547+2*(0.152547)+7.707962) = **0.1224649**

Group 5:

r = 1/(0.152547+2*(0.152547)+8.668884) = **0.1095707**

4 Comparing Returns to Scale Estimates from 1955 with Updated 1970 Data

Nerlove's returns to scale results were based on 1955 data for 145 electric utility companies in the United States. These data have been updated to 1970 and were subsequently by Christensen and Greene (1976). In this exercise, you compare returns-to-scale estimates based on the 1955 and the 1970 data and then evaluate the Christensen-Greene finding that by 1970 the bulk of electricity generation in the United States came from firms operating very near the bottom of their average cost curves. The 1970 data are presented in data file called UPDATE. The 1970 data sample is smaller, consisting of 99 observations, and like the data in NERLOV, the observations are ordered by size of firm, as measured by kilowatt hour output. The variables in the UPDATE data file include the original Christensen-Greene observation number (OBSNO), total costs in millions of 1970 dollars (COST70), millions of kilowatt hours of output (KWH70), the price of labor (PL70), the rental price index for capital (PK70), and the price index for fuels (PF70). (Notice that the numbers"70" have been added to the COST, KWH, PL, PK, and PF variables to distinguish these 1970 updated data from the Nerlove 1955 data.)

a. Using the 1970 updated data for 99 firms, construct the appropriate variables needed to estimate Nerlove's equation by least squares. In particular, for each of the 99 observations, generate the following variables: LNC70 = ln(COST70/PF70), LNY 70 = ln(KWH70), LNP170 = ln(PL70/PF70) and LNP270 = ln(PK70/PF70), where your just-constructed LNC70 is the same as lnC* in Nerlove's equation, LNY70 is ln y, LNP170 is ln p1, and LNP270 is ln p2. Compute the sample mean for KWH70, and compare it to the sample mean for KWH in Nerlove's 1955 data set, used above. On average, are firms generating larger amounts of electricity in 1970 than in 1955? What might you therefore expect in terms of returns-to-scale estimates for 1970 as compared to those for 1955? Why?

The sample mean for KWH70 is 8999.727 million kilowatt hours of output, comparing this to the Nerlov dataset, the sample mean for KWH50 is 2133.083 million kilowatt hours of output. This shows that on average, firms are generating larger amounts of electricity in 1970 than in 1955.

For the returns-to-scale estimates, I will expect these to be *lower* compared to those calculated using the 1955 dataset. The reason for this is that in the 1955 dataset, the utility companies whose output was the largest (those in group five) had the lowest returns to scale at ~ 0.94. The data essentially revealed that past a certain threshold of KWH of electricity generated, returns to scale will decline. Whether or not this persists for the 1970 data is examined in subsequent questions.

Note: the practicum assignment says the unit of measurement is billion for the 1955 dataset and million for the 1970 dataset. I believe this is a typo, so I am assuming they are both in millions, since in 1970 there surely was more electricity generated on average by plant compared to that in 1950.

b. Now estimate the parameters of the equation:

$$ln(C70) = \beta_0 + \beta_y(ln(Y70)) + \beta_1(ln(P170)) + \beta_2(ln(p270)) + \epsilon$$

by least squares and then construct a confidence interval for By, using a reasonable level of significance. Is the null hypothesis of constant returns to scale (By = 1) rejected? What does this imply concerning a test of the null hypothesis that returns to scale are constant? Using the relation between returns to scale r and By (By = (1/r)), calculate the implied estimate of returns to scale.Compare this result, based on the 1970 data, with that reported by Nerlove for his 1955 data (see Exercise 2, part (b), for a list of Nerlove's results). Are you surprised by these results? Why or why not?

Table 6: Model Five Summary Statistics

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-1.9250855	0.6394255	-3.010648	0.0033392
LNY70	0.8419840	0.0142489	59.091206	0.0000000
LNP170	-0.2963745	0.1236294	-2.397282	0.0184734
LNP270	-0.3472568	0.1111575	-3.124007	0.0023657

Constructing the confidence interval for B_y:

Formula:

$$CI = B_y \pm 1.96(SE(B_y))$$

$$CI = 0.84198 \pm 1.96(0.01425)$$

Confidence interval: (0.81405,0.86991) Using this confidence interval, evaluated at the 95% signficance we see that 1 does not lie within the range of values given, thus, the statistical decision is to reject the null hypothesis of constant returns to scale in favor of the alternative, that returns to scale are not constant.

Calculating the implied estimate returns to scale: Formula:

$$r = 1/(\beta_y)$$

$$r = 1/(0.84198) = 1.187677$$

Comparing this to Exercise 2, part (b) which found that the returns to scale from the corresponding least-squares regression model was 1.387563 the returns to scale in the model are slightly lower, though are still increasing as r > 1. This result does not surprise me, it makes sense that the same model specification will return a similar calculation for the returns to scale with most factors such as the structure of the data being held equal. A potential explanation for why the increasing returns to scale persist is that technology had improved over the 15+ year time frame which had elapsed, which allows for the returns to scale to keep up with the drastic increase in mean KWH produced by the utility companies.

c. A slightly generalized version of Nerlove equation involves adding (ln(y70))^2 as a regressor.

$$ln(C70) = \beta_0 + \beta_y ln(ln(Y70)) + \beta_{yy} * ln(Y70))^2 + \beta_1 ln(P170) + \beta_2 ln(P270) + \epsilon$$

Note that this estimating equation cannot be derived from the Cobb-Douglas production function

$$y = A * x_1^{\alpha_1} * x_2^{\alpha_2} * x_3^{\alpha_3}$$

but has the advantage of permitting returns to scale to vary with the level of output. In particular, in the above equation, returns to scale can be shown to equalr = 1/(By + 2 × Byy × ln y). Note also that the equation in part (b) of this exercise is a special case of the expanded equation here, being valid if and only if Byy = 0. Using the 1970 data, estimate by least squares the parameters of the above expanded equation. Then, based on a reasonable level of significance, test the null hypothesis that returns to scale do not vary with the level of output, that is, test the null hypothesis that Byy = 0 against the alternative hypothesis that Byy != 0. Next test the joint null hypothesis that returns to scale are constant, that is, that Byy = 0 andBy = 1, against the alternative hypothesis thatByy != 0 , By != 1. Interpret these two different test results. Are they mutually consistent?

Table 7: Model Six Summary Statistics

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-0.9328529	0.4864396	-1.917716	0.0581828
LNY70	0.3140576	0.0602348	5.213893	0.0000011
SQLNY70	0.0370255	0.0041592	8.902125	0.0000000
LNP170	-0.1463768	0.0930858	-1.572492	0.1191964
LNP270	-0.4623545	0.0833219	-5.549016	0.0000003

Table 8: Joint Null Hypothesis:

Res.Df	Df	F	Pr(>F)
96	NA	NA	NA
94	2	103.8991	0

Testing the null hypothesis that returns to scale do not vary with the level of output: Examining the regression output for model six, we see that t = 8.902, because |t| > 2, the statistical decision is to reject the null hypothesis that B__yy = 0 in favor of the alternative that B__yy != 0.

The results from the Joint Null Hypothesis test given above show that we reject the null hypothesis that B__yy = 0 and B__y = 1 in favor of the alternative hypothesis that B__yy != 0 and B__y != 1. Justification, P < 0.05. This gives statistical support to the result that the firms in this dataset do not exhibit constant returns to scale. Examining whether or not these results are mutually consistent, both come to the same conclusion of rejecting the null hypothesis of B__yy = 0 and thus they both support the conclusion of non-constant returns to scale.

d. Next, calculate the implied range of returns to scale by splitting the 1970 sample into five groups, ordered by size, where the first four groups consist of 20 firms each and the last group has only 19 firms. Estimate by least squares the parameters of the equation in part (b) separately for each of the five groups. For each group, compare the returns-to-scale estimates based on 1970 data with those reported by Nerlove and based on 1955 data, namely, 2.92, 2.24, 1.97, 1.84, and 1.69.

Note: Beta coefficients and respective standard errors in this table are presented in the same order as in the model specification. B_0, B_y, B_1, B_2

Table 9: Summary Statistics for each Utility Company Group (1970)

Group	Coefficients	StandardError	Rsquared
1	-1.766, 0.6811, -0.1993, -0.1257	(1.3552), (0.0407), (0.2654), (0.1989)	0.9526
2	-3.1909, 0.701, 0.172, -0.7553	(1.521), (0.1441), (0.2369), (0.1952)	0.8231
3	-5.1217, 1.0736, -0.0261, -0.7471	(1.0313), (0.0957), (0.1058), (0.138)	0.9113
4	-1.7471, 0.897, -0.3475, -0.7569	(2.6273), (0.2286), (0.2178), (0.1864)	0.8505
5	-4.6455, 0.9489, 0.1123, -0.8722	(1.4694), (0.1141), (0.2022), (0.2253)	0.8575

Computing the returns to scale for each utility company size (denoted group 1-5 in the previous table summary statistics) Formula: Note: B^k, indicates the beta coefficient for each group, k = 1,. . . 5

$$r = 1/(\beta_y^k)$$

$$r_1 = 1/(0.6811) = \underline{1.468213}$$

$$r_2 = 1/(0.701) = \underline{1.426534}$$

$$r_3 = 1/(1.0736) = \underline{0.9314456}$$

$$r_4 = 1/(0.897) = \underline{1.114827}$$

$$r_5 = 1/(0.9489) = \underline{1.053852}$$

Comparing these results with those from the previous dataset, whose returns to scale in groups 1-5 are: 2.49, 1.51, 1.06, 1.09, and 0.96, whereas the returns in this dataset in groups 1-5 are: 1.47, 1.43, 0.93, 1.11, 1.05. Comparing these, we see very similar patterns in that they are higher for the lower output firms compared to the high output firms. A unexplainable difference that might be worth looking into is why group three in the new dataset exhibits diminishing returns to scale.

e. Finally, how might one best evaluate the Christensen-Greene finding that by 1970 the bulk of U.S. electricity generation was being produced by firms operating ”very close” to the bottom of their average cost curves? Do you agree or disagree with Christensen and Greene? Why

One way to evaluate this finding would be to establish functions that give the cost of electricity generation, then divide these by quantity to give the average cost functions. Using these average cost functions, we would then determine the equilibrium in the electricity markets to determine the market-clearing price and quantity for electricity production. Within this dataset we have completed a part of this analysis, and we can regress the cost variable (not ln(cost)) on all inputs to production to determine the cost, and divide this by quantity produced to find the average cost. *A key consideration however, is that these costs are not representative of the entire cost faced by the utility company* there are other costs such as rent that need to be considered for us to develop a true average cost function. After developing this and determining the market equilibrium, calculating the share of firms that operate at the bottom of average cost firms can either confirm or reject the Christensen and Green finding.

Based on the analysis conducted here, the fact that utility companies, at higher levels of output begin to exhibit diminishing marginal returns (to just LNKWH, as well as to LNKWH LNP1 and LNP2) constitutes a strong reason to believe that firms are operating at the bottom of their average cost functions.

```r
library(knitr)
# install the tidyverse library (do this once) install.packages('tidyverse')
library(tidyverse)
install.packages("car", repos = "http://cran.us.r-project.org")
library(openxlsx)
library(car)

knitr::opts_chunk$set(echo = FALSE, message = FALSE, warning = FALSE, fig.width = 4,
    fig.height = 4, tidy = TRUE)
# chunk of code to load data, packages etc.

# get data
library(readxl)
nerlov <- read_excel("nerlov.xlsx")

# use the data transformation facilities of R to create new variables within
# the nerlov dataset
nerlov$LNCP3 <- log(nerlov$COSTS/nerlov$PF)
nerlov$LNP13 <- log(nerlov$PL/nerlov$PF)
nerlov$LNP23 <- log(nerlov$PK/nerlov$PF)
nerlov$LNKWH <- log(nerlov$KWH)

# printing the data series. Not necessary to print the entire thing; we can see
# that the observations are ordered by size of output if we use the head and
# tail functions to print the first five and last five observations. If they
# are in order from smallest to largest output, LNKWH will be small for the
# head function, and large for the tail function. I think this would be a
# better way to show what the question asks.
print("First five observations of the dataframe")
head(nerlov)
print("Last five observations of the dataframe")
tail(nerlov)

file_path <- getwd()
write.xlsx(nerlov, file_path, sheetName = "Nerlov_written", rowNames = FALSE)
# regression- models begin at model 1 with summary tables sum 1, and continue
# until the end of the practicum
model_1 <- lm(LNCP3 ~ LNKWH + LNP13 + LNP23, data = nerlov)
sum_1 <- summary(model_1)
kable(sum_1$coefficients, caption = "Summary Statistics for Model One")


# extract the residuals from the model summary statistics
model1_res <- model_1$residuals
# extract the log of output
lnoutput <- nerlov$LNKWH

# create a nice looking plot:
ggplot() + geom_point(aes(x = lnoutput, y = model1_res)) + theme_minimal() + xlab("LNKWH") +
    ylab("Model 1 Residuals") + ggtitle("Plot of LNKWH versus Residuals")


# another interesting plot is to illustrate (for all firms the fitted values
```

```r
# versus the actual values) this helps us understand for which firms the model
# underpredicts and for which firms the model overpredicts
library(stringr)
ggplot() + ggtitle(str_wrap("Plot of Observed values versus the Fitted for Utility Companies",
    width = 30)) + xlab("Firm Size") + ylab("Output Level") + geom_point(aes(x = nerlov$Obs,
    y = nerlov$LNCP3, color = "Observed"), size = 1) + geom_point(aes(x = nerlov$Obs,
    y = model_1$fitted.values, color = "Fitted Values"), size = 1) + labs(color = "Data Type") +
    scale_color_manual(values = c("blue", "red")) + theme_minimal()

# examining the sample correlation:
cor(model1_res, nerlov$LNKWH)


# first we need to group our data, putting our 145 observations into five
# different groups to distinguish by size: create a copy of the dataset
nerlov2 <- nerlov
# use integer division to create groups. Note: group 1 has the smallest output,
# and it increases until group 5- with the largest output
nerlov2$group <- (nerlov2$Obs - 1)%/%29 + 1

# regression:
model_list <- list()
summary_list <- list()
for (group in 1:5) {
    response_var <- "LNCP3"
    independent_vars <- c("LNKWH", "LNP13", "LNP23")
    formula <- as.formula(paste(response_var, "~", paste(independent_vars, collapse = " + ")))

    # Run the regression for the specific group
    model <- lm(formula, data = nerlov2[nerlov2$group == group, ])
    model_summary <- summary(model)
    model_list[[as.character(group)]] <- list(model = model, summary = model_summary)

    summary_stats <- list(coefficients = coef(model), rsquared = model_summary$r.squared,
        std_errors = model_summary$coefficients[, "Std. Error"])

    summary_list[[as.character(group)]] <- summary_stats
}

# create a dataframe of summary statistics to be reported
coefficients_df <- data.frame(Group = 1:5, Coefficients = sapply(summary_list, function(x) paste(round(x
    4), collapse = ", ")), StandardError = sapply(summary_list, function(x) paste0("(",
    round(x$std_errors, 4), ")", collapse = ", ")), Rsquared = round(sapply(summary_list,
    function(x) x$rsquared), 4))

# Print the table using kable
kable(coefficients_df, caption = "Summary Statistics for each Utility Company Group")

# creating a model in which the intercept and the LNKWH (BY) variable vary by
# group
nerlov2$groupfac <- as.factor(nerlov2$group)  # creating a column that makes group a factor variable, t
model_2 <- lm(LNCP3 ~ LNP13 + LNP23 + groupfac + (groupfac * LNKWH), data = nerlov2)
sum_2 <- summary(model_2)
```

```r
kable(coefficients(sum_2), caption = "Summary Statistics for Model Three")




# now the regression has a nonlinear specification there is a term LNKWH^2
# creating the variable:
nerlov2$SQLNKWH <- nerlov2$LNKWH^2

# running the model
model_4 <- lm(LNCP3 ~ LNKWH + SQLNKWH + LNP13 + LNP23, data = nerlov2)
sum_4 <- summary(model_4)
kable(sum_4$coefficients, caption = "Summary Statistics for Model Four")

# performing the f tests
hyp_1 <- linearHypothesis(model_4, c("LNKWH=1", "SQLNKWH=0"), white.adjust = "hc1")
kable(hyp_1, caption = "Joint Null Hypothesis:")



# pulling median values for KWH for each of the five subgroups
median_by_group <- nerlov2 %>%
    group_by(group) %>%
    summarise(median_LNKWH = median(LNKWH))


# load the dataset- update:
library(readxl)
update <- read_excel("update.xlsx")

# using the data transformation facilities of R, generate the variables
# required to estimate the parameters, now for the Christensen dataset:
update$LNC70 <- log(update$COST70)
update$LNY70 <- log(update$KWH70)
update$LNP170 <- log(update$PL70/update$PF70)
update$LNP270 <- log(update$PK70/update$PF70)

meanKWH70 <- mean(update$KWH70)
meanKWH50 <- mean(nerlov$KWH)




# regression
model_5 <- lm(LNC70 ~ LNY70 + LNP170 + LNP270, data = update)
sum_5 <- summary(model_5)
kable(coefficients(sum_5), caption = "Model Five Summary Statistics")

# create a new variable in the update dataset for the KWH^2
update$SQLNY70 <- update$LNY70^2
# run the regression:
model_6 <- lm(LNC70 ~ LNY70 + SQLNY70 + LNP170 + LNP270, data = update)
sum_6 <- summary(model_6)
kable(coefficients(sum_6), caption = "Model Six Summary Statistics")
```

```r
# running the linear hypothesis:
hyp_2 <- linearHypothesis(model_6, c("LNY70=1", "SQLNY70=0"), white.adjust = "hc1")
kable(hyp_2, caption = "Joint Null Hypothesis:")

# this requires the same process that was used in question three: create a copy
# of the dataset
update2 <- update
# use integer division to create groups. Note: group 1 has the smallest output,
# and it increases until group 5- with the largest output
update2$group <- (update2$Obs - 1)%/%20 + 1

# first splitting the dataset

model_list2 <- list()
summary_list2 <- list()
for (group in 1:5) {
    response_var <- "LNC70"
    independent_vars <- c("LNY70", "LNP170", "LNP270")
    formula <- as.formula(paste(response_var, "~", paste(independent_vars, collapse = " + ")))

    # Run the regression for the specific group
    model <- lm(formula, data = update2[update2$group == group, ])
    model_summary <- summary(model)
    model_list2[[as.character(group)]] <- list(model = model, summary = model_summary)

    summary_stats <- list(coefficients = coef(model), rsquared = model_summary$r.squared,
        std_errors = model_summary$coefficients[, "Std. Error"])

    summary_list2[[as.character(group)]] <- summary_stats
}

# create a dataframe of summary statistics to be reported
coefficients_df <- data.frame(Group = 1:5, Coefficients = sapply(summary_list2, function(x) paste(round
    4), collapse = ", ")), StandardError = sapply(summary_list2, function(x) paste0("(",
    round(x$std_errors, 4), ")", collapse = ", ")), Rsquared = round(sapply(summary_list2,
    function(x) x$rsquared), 4))

# Print the table using kable
kable(coefficients_df, caption = "Summary Statistics for each Utility Company Group (1970)")
```

Obs	ORDER	COSTS	KWH	PL	PF	PK
1	101	0.082	2	2.08999991	17.8999996	183
2	102	0.66100001	3	2.04999995	35.0999985	174
3	103	0.99000001	4	2.04999995	35.0999985	171
4	104	0.315	4	1.83000004	32.2000008	166
5	105	0.197	5	2.11999989	28.6000004	233
6	106	0.098	9	2.11999989	28.6000004	195
7	107	0.949	11	1.98000002	35.5	206
8	108	0.67500001	13	2.04999995	35.0999985	150
9	109	0.52499998	13	2.19000006	29.1000004	155
10	110	0.50099999	22	1.72000003	15	188
11	111	1.19400001	25	2.08999991	17.8999996	170
12	112	0.67000002	25	1.67999995	39.7000008	167
13	113	0.34900001	35	1.80999994	22.6000004	213
14	114	0.42300001	39	2.29999995	23.6000004	164
15	115	0.50099999	43	1.75	42.7999992	170
16	116	0.55000001	63	1.75999999	10.3000002	161
17	117	0.79500002	68	1.98000002	35.5	210
18	118	0.66399997	81	2.28999996	28.5	158
19	119	0.70499998	84	2.19000006	29.1000004	156
20	120	0.903	73	1.75	42.7999992	176
21	121	1.50399995	99	2.20000005	36.2000008	170
22	122	1.61500001	101	1.65999997	33.4000015	192
23	123	1.12699997	119	1.91999996	22.5	164
24	124	0.71799999	120	1.76999998	21.2999992	175
25	125	2.41400003	122	2.08999991	17.8999996	180
26	126	1.13	130	1.82000005	38.9000015	176
27	127	0.99199998	138	1.79999995	20.2000008	202
28	128	1.55400002	149	1.91999996	22.5	227
29	129	1.22500002	196	1.91999996	29.1000004	186
30	201	1.56500006	197	2.19000006	29.1000004	183
31	202	1.93599999	209	1.91999996	22.5	169
32	203	3.15400004	214	1.51999998	27.5	168
33	204	2.59899998	220	1.91999996	22.5	164
34	205	3.2980001	234	2.20000005	36.2000008	164
35	206	2.44099998	235	2.1099999	24.3999996	170
36	207	2.0309999	253	1.91999996	22.5	158
37	208	4.66599989	279	2.04999995	35.0999985	177
38	209	1.83399999	290	1.65999997	33.4000015	195
39	210	2.07200003	290	1.79999995	20.2000008	176
40	211	2.03900003	295	1.76999998	21.2999992	188
41	212	3.398	299	1.70000005	26.8999996	187
42	213	3.08299994	324	2.04999995	35.0999985	152
43	214	2.3440001	333	2.19000006	29.1000004	157
44	215	2.38199997	338	1.85000002	24.6000004	163
45	216	2.65700006	353	2.19000006	29.1000004	143
46	217	1.70500004	353	2.13000011	10.6999998	167
47	218	3.23000002	416	1.53999996	26.2000008	217
48	219	5.04899979	420	1.51999998	27.5	144
49	220	3.81399989	456	2.08999991	30	178

50	221	4.57999992	484	1.75	42.7999992	176
51	222	4.3579998	516	2.29999995	23.6000004	167
52	223	4.71400023	550	2.04999995	35.0999985	158
53	224	4.35699987	563	2.31999993	31.8999996	162
54	225	3.91899991	566	2.30999994	33.5	198
55	226	3.44199991	592	1.91999996	22.5	164
56	227	4.89799976	671	2.04999995	35.0999985	164
57	228	3.58400011	696	1.75999999	10.3000002	161
58	229	5.53499985	719	1.70000005	26.8999996	174
59	301	4.40600014	742	2.03999996	20.7000008	157
60	302	4.28900003	795	2.24000001	26.5	185
61	303	6.73099995	800	1.70000005	26.8999996	157
62	304	6.89499998	808	1.67999995	39.7000008	203
63	305	5.11199999	811	2.28999996	28.5	178
64	306	5.14099979	855	2	34.2999992	183
65	307	5.71999979	860	2.30999994	33.5	168
66	308	4.69099998	909	1.45000005	17.6000004	196
67	309	6.83199978	913	1.70000005	26.8999996	166
68	310	4.8130002	924	1.75999999	10.3000002	172
69	311	6.75400019	984	1.70000005	26.8999996	158
70	312	5.12699986	991	2.08999991	30	174
71	313	6.38800001	1000	1.54999995	28.2000008	225
72	314	4.50899982	1098	2.1099999	24.3999996	168
73	315	7.18499994	1109	2.04999995	35.0999985	177
74	316	6.80000019	1118	2.29999995	23.6000004	161
75	317	7.74300003	1122	2.19000006	29.1000004	162
76	318	7.96799994	1137	2.03999996	20.7000008	158
77	319	8.8579998	1156	2.30999994	33.5	176
78	320	8.5880003	1166	1.70000005	26.8999996	183
79	321	6.44899988	1170	2.04999995	35.0999985	166
80	322	8.48799992	1215	2.19000006	29.1000004	164
81	323	8.87699986	1279	2	34.2999992	207
82	324	10.2740002	1291	2.31999993	31.8999996	175
83	325	6.02400017	1290	1.54999995	28.2000008	225
84	326	8.25800037	1331	2.13000011	30	178
85	327	13.3760004	1373	2.20000005	36.2000008	157
86	328	10.6899996	1420	2.20000005	36.2000008	138
87	329	8.30799961	1474	1.85000002	24.6000004	163
88	401	6.08199978	1497	1.75999999	10.3000002	168
89	402	9.2840004	1545	1.79999995	20.2000008	158
90	403	10.8789997	1649	2.31999993	31.8999996	177
91	404	8.47700024	1668	1.79999995	20.2000008	170
92	405	6.87699986	1782	2.13000011	10.6999998	183
93	406	15.1059999	1831	1.98000002	35.5	162
94	407	8.03100014	1833	1.75999999	10.3000002	177
95	408	8.08199978	1838	1.45000005	17.6000004	196
96	409	10.8660002	1787	2.24000001	26.5	164
97	410	8.59599972	1918	1.69000006	12.8999996	158
98	411	8.67300034	1930	1.80999994	22.6000004	157
99	412	15.4370003	2028	2.1099999	24.3999996	163

100	413	8.21100044	2057	1.75999999	10.3000002	161
101	414	11.9820004	2084	1.76999998	21.2999992	156
102	415	16.6739998	2226	2	34.2999992	217
103	416	12.6199999	2304	2.29999995	23.6000004	161
104	417	12.9049997	2341	2.03999996	20.7000008	183
105	418	11.6149998	2353	1.69000006	12.8999996	167
106	419	9.3210001	2367	1.75999999	10.3000002	161
107	420	12.9619999	2451	2.03999996	20.7000008	163
108	421	16.9319992	2457	2.20000005	36.2000008	170
109	422	9.64799976	2507	1.75999999	10.3000002	174
110	423	18.3500004	2530	2.30999994	33.5	197
111	424	17.3330002	2576	1.91999996	22.5	162
112	425	12.0150003	2607	1.75999999	10.3000002	155
113	426	11.3199997	2870	1.75999999	10.3000002	167
114	427	22.3369999	2993	2.30999994	33.5	176
115	428	19.0349998	3202	2.29999995	23.6000004	170
116	429	12.2049999	3286	1.61000001	17.7999992	183
117	501	17.0779991	3312	1.67999995	28.7999992	190
118	502	25.5279999	3498	2.08999991	30	170
119	503	24.0209999	3538	2.08999991	30	176
120	504	32.1969986	3794	2.04999995	35.0999985	159
121	505	26.6520004	3841	2.28999996	28.5	157
122	506	20.1639996	4014	2.1099999	24.3999996	161
123	507	14.132	4217	1.52999997	18.1000004	172
124	508	21.4099998	4305	2.1099999	24.3999996	203
125	509	23.2439995	4494	2.03999996	20.7000008	167
126	510	29.8449993	4764	2.19000006	29.1000004	195
127	511	32.3180008	5277	1.91999996	29.1000004	161
128	512	21.9880009	5283	2.03999996	20.7000008	159
129	513	35.2290001	5668	2.1099999	24.3999996	177
130	514	17.4669991	5681	1.75999999	10.3000002	157
131	515	22.8279991	5819	1.78999996	18.5	196
132	516	33.1539993	6000	2.1099999	24.3999996	183
133	517	32.2280006	6119	1.53999996	26.2000008	189
134	518	34.1679993	6136	1.91999996	22.5	160
135	519	40.5940018	7193	2.11999989	28.6000004	162
136	520	33.3540001	7886	1.61000001	17.7999992	178
137	521	64.5419998	8419	2.31999993	31.8999996	199
138	522	41.237999	8642	2.24000001	26.5	182
139	523	47.993	8787	2.30999994	33.5	190
140	524	69.8779984	9484	2.1099999	24.3999996	165
141	525	44.894001	9956	1.67999995	28.7999992	203
142	526	67.1200027	11477	2.24000001	26.5	151
143	527	73.0500031	11796	2.11999989	28.6000004	148
144	528	139.421997	14359	2.30999994	33.5	212
145	529	119.939003	16719	2.29999995	23.6000004	162

LNCP3	LNP13	LNP23	LNKWH
-5.3858367	-2.14763667	2.32468546	0.69314718
-3.97220251	-2.84036132	1.60085421	1.09861229
-3.56825141	-2.84036132	1.58346247	1.38629436
-4.62714912	-2.86765049	1.64002131	1.38629436
-4.9779583	-2.6019907	2.09763172	1.60943791
-5.67619456	-2.6019907	1.91959283	2.19722458
-3.62187918	-2.88643584	1.75834347	2.39789527
-3.95124366	-2.84036132	1.45243421	2.56494936
-4.01509525	-2.58683662	1.67268693	2.56494936
-3.3991994	-2.16572589	2.52839176	3.09104245
-2.70749167	-2.14763667	2.25099775	3.21887582
-4.08182875	-3.16255744	1.43664261	3.21887582
-4.17063326	-2.52462311	2.24334224	3.55534806
-4.02162981	-2.32833763	1.9386197	3.66356165
-4.44768729	-3.1969223	1.37926035	3.76120012
-2.92998089	-1.76683011	2.74926045	4.14313473
-3.79894584	-2.88643584	1.77757483	4.21950771
-3.75937725	-2.52135229	1.71269095	4.39444915
-3.72029569	-2.58683662	1.67911782	4.4308168
-3.85857081	-3.1969223	1.41394591	4.29045944
-3.18093095	-2.80060176	1.5467393	4.59511985
-3.02922098	-3.00173836	1.74893943	4.61512052
-2.9939561	-2.46119015	1.98635112	4.77912349
-3.38999275	-2.4877275	2.10607894	4.78749174
-2.00351555	-2.14763667	2.30815616	4.80402104
-3.53877666	-3.06215776	1.50948971	4.86753445
-3.01371483	-2.417896	2.30258506	4.92725369
-2.67268304	-2.46119015	2.31143471	5.00394631
-3.16779732	-2.71841302	1.85500849	5.27811466
-2.92285233	-2.58683662	1.83874797	5.28320373
-2.45289133	-2.46119015	2.01638341	5.34233425
-2.1655145	-2.89547568	1.80977797	5.36597602
-2.15838856	-2.46119015	1.98635112	5.39362755
-2.39574289	-2.80060176	1.51080729	5.45532112
-2.30217533	-2.44789522	1.94121532	5.45958551
-2.40498708	-2.46119015	1.94907972	5.53338949
-2.01789894	-2.84036132	1.61794865	5.63121178
-2.90205658	-3.00173836	1.76444361	5.66988092
-2.2771683	-2.417896	2.16480135	5.66988092
-2.34624753	-2.4877275	2.17773493	5.68697536
-2.06893925	-2.76149799	1.93898234	5.70044357
-2.43229796	-2.84036132	1.46567943	5.78074352
-2.51887927	-2.58683662	1.68550762	5.80814249
-2.334806	-2.58756081	1.89100374	5.8230459
-2.3935405	-2.58683662	1.59210644	5.86646806
-1.83667859	-1.61412169	2.74775009	5.86646806
-2.0932773	-2.83397705	2.11413791	6.03068526
-1.69499584	-2.89547568	1.65562729	6.04025471
-2.0625189	-2.66403336	1.78058617	6.12249281

-2.2348391	-3.1969223	1.41394591	6.18208491
-1.68923354	-2.32833763	1.95674708	6.24610677
-2.00766423	-2.84036132	1.50439395	6.30991828
-1.99082228	-2.62103884	1.62499034	6.33327963
-2.14570894	-2.67429794	1.77672159	6.33859408
-1.87746264	-2.46119015	1.98635112	6.38350663
-1.96937418	-2.84036132	1.54166534	6.50876914
-1.05566439	-1.76683011	2.74926045	6.54534966
-1.58103473	-2.76149799	1.86692903	6.57786136
-1.54716646	-2.31718395	2.02611207	6.60934924
-1.82109112	-2.47066886	1.94321109	6.67834211
-1.38540256	-2.76149799	1.76411953	6.68461173
-1.7505547	-3.16255744	1.63185477	6.69456206
-1.71831337	-2.52135229	1.83187946	6.69826805
-1.89789776	-2.84199815	1.67434082	6.75110147
-1.76757667	-2.67429794	1.61241854	6.75693239
-1.32225315	-2.49633533	2.41021574	6.81234509
-1.37050885	-2.76149799	1.81986152	6.81673588
-0.76082328	-1.76683011	2.81535056	6.82871207
-1.38199132	-2.76149799	1.77046876	6.8916259
-1.76667672	-2.66403336	1.75785792	6.89871453
-1.48490077	-2.9010671	2.0767784	6.90775528
-1.68850776	-2.44789522	1.92938086	7.00124562
-1.58620558	-2.84036132	1.61794865	7.01121399
-1.24432409	-2.32833763	1.92015764	7.01929665
-1.32394897	-2.58683662	1.71685815	7.02286809
-0.95470022	-2.31718395	2.0324613	7.03614849
-1.33022446	-2.67429794	1.65893856	7.05272105
-1.14176036	-2.76149799	1.91735988	7.06133437
-1.69427603	-2.84036132	1.5537867	7.06475903
-1.2320848	-2.58683662	1.72912824	7.10249936
-1.35168169	-2.84199815	1.79757346	7.1538338
-1.13298955	-2.62103884	1.70217998	7.16317239
-1.54357049	-2.9010671	2.0767784	7.1623975
-1.29001491	-2.64507535	1.78058617	7.19368582
-0.99559705	-2.80060176	1.46718667	7.22475341
-1.21975045	-2.80060176	1.33819455	7.25841215
-1.0855276	-2.58756081	1.89100374	7.29573507
-0.52681036	-1.76683011	2.79182007	7.31121838
-0.77739011	-2.417896	2.05691239	7.34277919
-1.0757717	-2.62103884	1.71354373	7.40792432
-0.868326	-2.417896	2.13011579	7.41938058
-0.44206123	-1.61412169	2.83924243	7.48549161
-0.85444068	-2.88643584	1.51806364	7.51261754
-0.24883484	-1.76683011	2.84400582	7.51370925
-0.77825958	-2.49633533	2.41021574	7.5164333
-0.89150607	-2.47066886	1.82272169	7.48829352
-0.40593034	-2.03249872	2.50536775	7.55903826
-0.95773513	-2.52462311	1.93829588	7.56527528
-0.45781587	-2.44789522	1.89916708	7.61480536

-0.22666914	-1.76683011	2.74926045	7.62900389
-0.57530148	-2.4877275	1.99114897	7.6420444
-0.72129472	-2.84199815	1.84475202	7.70796153
-0.62596388	-2.32833763	1.92015764	7.74240202
-0.47251892	-2.31718395	2.17935242	7.75833347
-0.10492994	-2.03249872	2.56076653	7.76344639
-0.09987398	-1.76683011	2.74926045	7.76937861
-0.46811175	-2.31718395	2.06361646	7.80425138
-0.75985386	-2.80060176	1.5467393	7.80669637
-0.0653933	-1.76683011	2.82691139	7.8268421
-0.60191584	-2.67429794	1.77165829	7.83597458
-0.2609031	-2.46119015	1.97408103	7.85399309
0.15401198	-1.76683011	2.7112812	7.86595541
0.09442713	-1.76683011	2.7858499	7.96206731
-0.40530095	-2.67429794	1.65893856	8.00403151
-0.21496735	-2.32833763	1.97455171	8.07153089
-0.37735272	-2.40296423	2.33028774	8.0974263
-0.52258433	-2.8415816	1.88664871	8.10530752
-0.1614215	-2.66403336	1.73460106	8.15994666
-0.22226894	-2.66403336	1.76928661	8.17131687
-0.08632785	-2.84036132	1.51070312	8.24117615
-0.06703988	-2.52135229	1.70634172	8.25348803
-0.1906843	-2.44789522	1.88682125	8.29754353
-0.24747023	-2.47064424	2.25158252	8.34687925
-0.13072502	-2.44789522	2.11862286	8.36753242
0.11591327	-2.31718395	2.08786008	8.41049845
0.02527911	-2.58683662	1.90226137	8.46884293
0.10488619	-2.71841302	1.71066618	8.57111303
0.06036315	-2.31718395	2.03877047	8.5722494
0.36728649	-2.44789522	1.98156662	8.6425916
0.52816942	-1.76683011	2.72410189	8.64488255
0.21021708	-2.33555513	2.36034393	8.6688837
0.30658024	-2.44789522	2.01490304	8.69951475
0.20707622	-2.83397705	1.97598758	8.71915396
0.4177742	-2.46119015	1.96165851	8.72192834
0.35021358	-2.6019907	1.7341896	8.88086361
0.62797929	-2.40296423	2.30258514	8.97284431
0.70471017	-2.62103884	1.83069883	9.03824634
0.4422154	-2.47066886	1.92686195	9.06438932
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1.05216772	-2.44789522	1.91136236	9.15736145
0.44392882	-2.8415816	1.95283062	9.20593066
0.92933737	-2.47066886	1.7401351	9.34810031
0.93773745	-2.6019907	1.64380554	9.37551577
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1.62573658	-2.32833763	1.92634961	9.72430108